

Interferometric length metrology for the dimensional control of ultra-stable Ring Laser Gyroscopes

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Abstract. We present the experimental test of a method for controlling the absolute length of the diagonals of square ring laser gyroscopes. The purpose is to actively stabilize the ring cavity geometry and to enhance the rotation sensor stability in order to reach the requirements for the detection of the relativistic Lense-Thirring effect with a ground-based array of optical gyroscopes. The test apparatus consists of two optical cavities 1.32 m in length, reproducing the features of the ring cavity diagonal resonators of large frame He-Ne ring laser gyroscopes. The proposed measurement technique is based on the use of a single diode laser, injection locked to a frequency stabilized He-Ne/Iodine frequency standard, and a single electro-optic modulator. The laser is modulated with a combination of three frequencies allowing to lock the two cavities to the same resonance frequency and, at the same time, to determine the cavity Free Spectral Range (FSR). We obtain a stable lock of the two cavities to the same optical frequency reference, providing a length stabilization at the level of 1 part in 10^{11} , and the determination of the two FSRs with a relative precision of $\sim 2 \cdot 10^{-7}$. This is equivalent to an error of 500 nm on the absolute length difference between the two cavities.

1. Introduction

Large frame optical gyroscopes based on He-Ne ring laser technology are presently the most sensitive devices for measuring inertial rotational motion [1, 2]. The use of these systems has been considered in the past for several fundamental physics applications including experimental tests of fundamental symmetries [3], axions detection [4], test of metric theories of gravitation [5, 6], and gravitational waves detectors [7]. Recent improvements in the stability and resolution of this kind of devices fostered the GINGER (Gyroscope IN GEneral Relativity) project [8]. This experiment aims to develop a ground based array of large-frame laser gyroscopes (of at least 6 m in side) for the detection of the space-time geometry deformation induced by the Earth's rotation, also known as frame-dragging or Lense-Thirring effect.

The frequency difference f_S between the two opposite traveling beams in a laser gyroscope is given by the basic theory of the Sagnac effect:

$$f_S = \frac{4\vec{A} \cdot \vec{\Omega}}{\lambda P} \quad (1)$$

where \vec{A} is the ring cavity area vector enclosed by the ring laser optical path, P the cavity perimeter length, λ the laser wavelength and $\vec{\Omega}$ the frame angular velocity. Errors and instabilities in the geometrical scale factor $k_S = 4A/\lambda P$ affect directly the measured rotation rate. The present limit of the most stable devices is given by the slow deformations of the optical cavity induced by temperature, pressure and aging of materials. Several approaches have been followed in the last years in order to enhance the stability of the ring cavity geometry.

The best results have been obtained up to now by using monolithic structures operating in very well isolated, i.e. passively stabilized, laboratories. Today's best performing ring laser is the 4 m side G installed in a controlled area at the Geodetic Observatory of Wettzell, Germany [9]. It is a monolithic block of Zerodur with four mirrors mounted in optical contact with the cavity frame. It operated without any active control on the geometry until 2010. Since then, an active control of the cavity shape has been engaged. Such a control system stabilizes the ring laser optical frequency against an optical frequency reference by tuning the pressure inside a vacuum tank enclosing the whole monolithic cavity [10, 11]. In this way the cavity deformations induced by the variation of the pressure load on the four optically contacted mirrors are canceled to first order, with a consequent sensitive improvement of the long-term instrument stability.

For a ring laser in a ground-based laboratory, the Lense-Thirring term is of order of 1 part in 10^9 of the purely kinematic Sagnac term of equation 1. The direct measurement of this effect implies the stabilization of the scale factor with an accuracy of at least 1 part in 10^{10} . The stability of G is about one order of magnitude far from this target. A direct way to overcome the present limits is to increase the sensitivity of the ring by enlarging the cavity dimensions. However, it is not feasible to have a larger monolithic block of Zerodur; an alternative solution is to develop an active control acting on the corner mirrors positions. Experimental studies in this direction have been made on the G-Pisa ring laser [12]. It has been shown that locking the laser emission frequency to a suitable optical frequency standard provides a well defined configuration of the laser parameters but it is not the ultimate solution to the problem of the cavity deformation. In fact, variations of the area vector remain uncontrolled and the residual relative variation of the cavity side-lengths, even at constant perimeter, introduce instabilities on the rotational signal by changing the backscattering phase. This last error source dominates the dynamics of small ring lasers (side lengths of about 1 m) and it is strongly reduced by increasing the gyroscope linear dimensions. For side length larger than 6 m, the geometrical stability sets the main limit to the system performances and can be controlled only by means of interferometric methods. The ideal instrument would be a cavity stabilized to the shape of a perfect square, where the four sides are perpendicular and of the same absolute length. with a root mean square amplitude ε_{rms} , Both G and G-Pisa are square ring laser gyroscopes. One advantage of this cavity shape on the triangular one is that the two diagonals can constitute additional degrees of freedom to constrain. As a fact, typical He-Ne ring laser mirrors having a reflectivity of 99.999% at 45 deg angle of incidence, still have a good reflectivity at normal incidence. This allows us to use the same external laser source providing the perimeter reference frequency for stabilizing the length of the two Fabry-Pérot cavities formed by the opposite mirrors of the ring. By performing a beam steering simulation of the ring cavity it can be shown [13] that, when the length of the two diagonals is locked to the same value, the perturbations to the mirror positions affect only quadratically the ring laser scale factor. In figure 1 it is shown the simulated relative variation of the scale

factor $k_S = 4A/(\lambda P)$ induced by a random displacement noise applied to the four mirrors centers of curvature. The position of each mirror has been changed along the three directions of space by three independent random processes with root-mean-square amplitude ε_{rms} ; the case of a cavity with equal absolute diagonal lengths is compared to the uncontrolled case and to the case where the two lengths are both stabilized better than 1 part on 10^{11} but differ one from the other by a quantity δD . Details are in the caption.

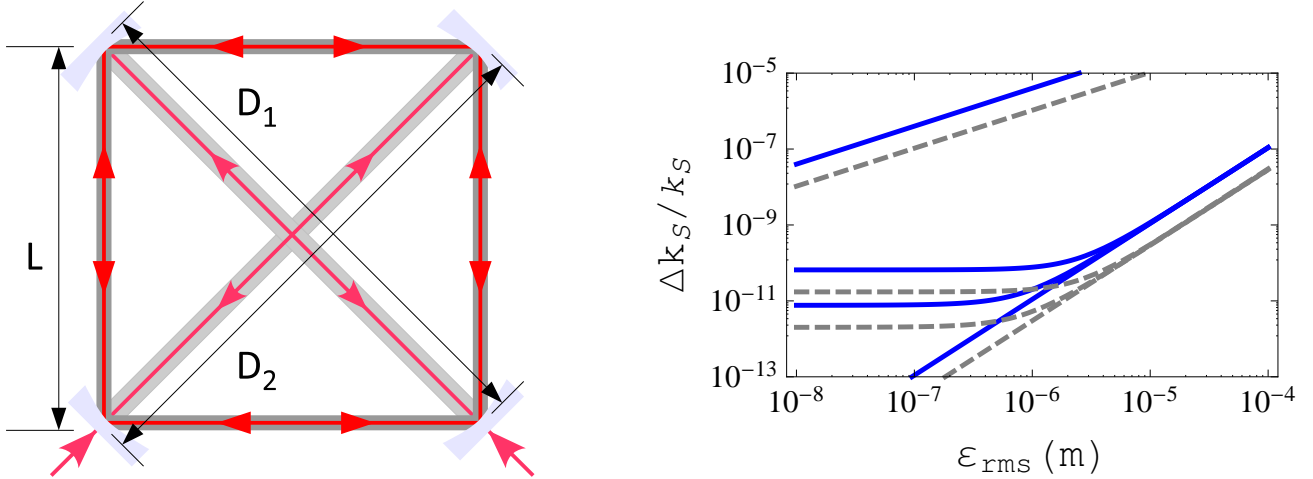


Figure 1. Left: Square ring laser gyroscope; diagonal Fabry-Pérot cavities, probed by an external laser source, are shown. Right: Relative scale factor variations induced in a square ring laser by a displacement noise with root mean square amplitude ε_{rms} acting on the four mirrors positions. Two cases with different side length L and mirrors curvature radius R are simulated: GP2 cavity, $L = 1.60$ m, $R = 4$ m (continuous blue line); GINGER cavity, $L = 6$ m, $R = 6$ m (gray dashed line). For each cavity, the different plotted curves correspond to the following cases (from top to bottom): free geometry; diagonal lengths stabilized but unbalanced of 10^{-6} m; diagonal lengths stabilized but unbalanced of 10^{-7} m; equal absolute diagonal lengths.

It is worth noting that even in the case of unbalances of the order of $1 \mu\text{m}$ the length stabilization constraint provides a high rejection of the mirrors position perturbations from the scale factor, with residual fluctuations at level of 1 part in 10^{10} . Motivated by this numerical results we designed a new ring laser gyroscope, called GP2 [14], dedicated to the optimization and stabilization of the geometrical form factor and, in particular, to the experimental implementation of the length stabilization of the diagonal cavities. It is a square resonator 1.6 m in side, equipped with a system of 6 piezoelectric actuators controlling the four mirrors positions. A special vacuum chamber encloses the four mirrors, the optical path along the ring cavity as well as the two diagonals.

In this paper we present an experimental technique to set equal the lengths of two linear Fabry-Pérot resonators formed by curved mirrors used typically as corner reflectors in He-Ne ring laser gyros, in order to simulate the ring cavity diagonals on an optical bench. The measurement of the cavity absolute length can be obtained with very high precision ($< 10^{-11}$) if the cavity resonance is locked to the frequency of an optical frequency standard and the integer number n of half-wavelengths contained between the two mirrors is unambiguously estimated. This last can be obtained by measuring the cavity FSR with a precision better than $1/n$. In the recent literature, particularly in the field of gravitational wave detectors, several techniques have been proposed to measure the FSR of long linear cavities. In particular, Araya

et al. [15] measured the absolute length of an approximately 300 m long Fabry-Pérot cavity by FSR measurement, using two EOM for frequency modulation and attaining a measurement uncertainty of 10^{-8} ; Aketagawa et al. [16] measured the FSR of an approximately 0.170 m long cavity using a single EOM with an uncertainty of 10^{-6} . For a more detailed list of developed techniques see also [17].

The strategy we adopted is based on the use of a single frequency-stabilized diode laser at 633 nm, phase modulated with a single EOM driven by a combination of three independent modulation frequencies. The first modulation provides the Pound-Drever-Hall signals for locking the two cavities to the same laser optical frequency. The second modulation creates a set of sidebands spaced by a harmonic of the FSR frequency. The third modulation is a small dithering of the second frequency for shifting the FSR resonance detection down to few tens kHz.

The optical reference frequency in the experiment is provided by a He-Ne laser, frequency stabilized on the saturated absorption line R-127 11-5 of Iodine. The accurate determination of the FSR is measured by locking a VCO (Voltage Controlled Oscillator) to the center of the cavity dynamic resonance and by counting the frequency with a counter whose stability is of 1 ppb.

In section 2 we give a theoretical description of the measurement technique. In section 3 we present the experimental apparatus including the laser source at 633 nm. Section 4 reports the discussion of the results concerning the stabilization of the cavity length, the estimate of the free-spectral-ranges of the two cavities and their absolute length difference.

2. Measurement principle

Let's consider a linear cavity formed by two concave spherical mirrors of radius R separated by a distance L . The frequency f_n of a TEM_{00} laser beam resonating in the longitudinal mode of order n is given by:

$$f_n = \left(\frac{v}{2L} \right) \left[n + \frac{1}{2\pi} (\Psi_R + \Phi_n) \right] \quad (2)$$

where v is the speed of light in the dielectric medium contained in the cavity, $\Psi_R = 2 \cos^{-1} [1 - (L/R)]$ the Gouy-phase correction, and Φ_n a frequency-dependent phase shift correction due to the dielectric coating of the mirrors. The cavity length L can be measured with an accuracy ultimately limited by the knowledge of the reference laser frequency if the cavity is locked to the laser and the integer number n is estimated together with Ψ_R and Φ_n . Ψ_R can be obtained with high accuracy from the knowledge of R , whereas the dielectric shift contribution term, depending on the mirror manufacturing properties, can be evaluated from the reflectivity curve of the mirrors [18]. However, for two cavities having the same kind of mirrors (i.e. produced in the same run of coating process) we can assume the two Φ_n to be equal.

For each cavity we require two error signals in order to lock the cavity resonance to the laser carrier frequency and, subsequently, to lock the phase modulation frequency to a harmonic of the FSR. The laser electric field injected into the two cavities has the form:

$$\begin{aligned} E_{in}(t) &= E_0 e^{i\{\omega_0 t + \alpha \sin \omega_A(t) + \beta \sin[(\omega_B + \Delta \sin \omega_C t)t]\}} \\ &= E_0 e^{i\omega_0 t} \left[(-1)^p \sum_p J_p(\alpha) e^{ip\omega_A t} \right] \left[(-1)^q \sum_q J_q(\beta) e^{iq(\omega_B + \Delta \sin \omega_C t)t} \right] \end{aligned} \quad (3)$$

here

- ω_0 is the optical frequency;
- $\Delta\omega_{cav} < \omega_A < FSR$ is the Pound Drever Hall modulation frequency for the carrier lock, where $\Delta\omega_{cav}$ is the cavity linewidth;
- $\omega_B = mFSR$ is the modulation frequency for the cavity dynamic resonance excitation; note that the error on the resonance frequency measurement nominally decreases with the integer m as m^{-1} ;
- $\omega_c \sim 30$ kHz is the frequency of the dithering applied to ω_B for the lock-in detection of the FSR resonance signal.

$J_p(\alpha)$, $J_q(\beta)$ are first kind Bessel functions, α and β are the modulation indexes for the modulations at ω_A and ω_B , and Δ is the frequency deviation of the dithering at ω_C . For small values of α and β , such that almost all the power is in the carrier and the first order sidebands, only the first sidebands of the modulated spectrum ($p=-1,0,1$ and $q=-1,0,1$) can be considered. Thus the incident field can be written in terms of 9 modes, i.e. the carrier optical frequency and two group of four sidebands, as shown in figure 2. Within this representation we can write:

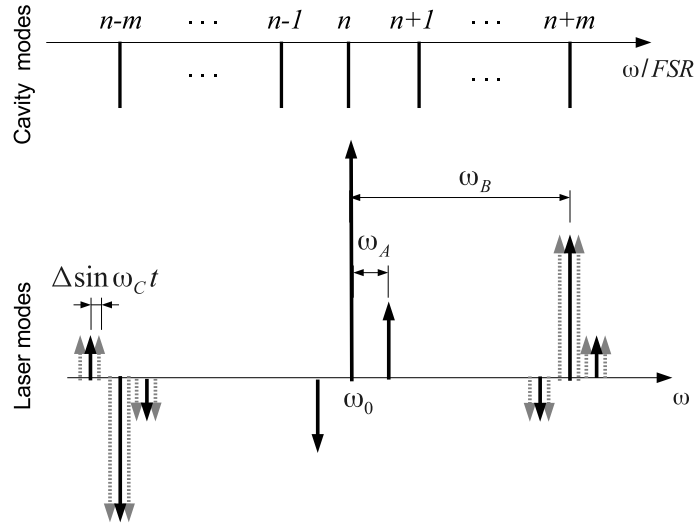


Figure 2. Schematic diagram of the cavity modes and the laser spectral structure. The frequency dithering at ω_C is not applied to the central pattern of three modes, but only to the six extreme modes.

$$E_{in}(t) = E_0 \sum_{k=1}^9 A_k e^{i\omega_k t},$$

where the 9 pairs of A_k , ω_k are defined in table A1 of Appendix. The reflected field $E_r(t)$ is obtained by applying the cavity reflection transfer function $R(\omega_k)$, defined in Appendix (equation A.1), to each of the k_{th} field components:

$$E_r(t) = E_0 \sum_{k=1}^9 A_k R(\omega_k) e^{i\omega_k t}.$$

The signal detected by the photodetector is proportional to the reflected field intensity $I_r \propto |E_r|^2$, i.e.:

$$I_r(t) \propto E_0^2 \sum_{k,k'=1}^9 A_k A_{k'} R(\omega_k) R^*(\omega_{k'}) e^{i(\omega_k - \omega_{k'})t},$$

where $R^*(\omega_k)$ is the conjugate of the cavity reflection transfer function.

If we select, by means of a phase sensitive detection technique, the components of the signal oscillating in phase with $\sin \omega_A t$ and $\sin \omega_C t$, see A.2 and A.3, we obtain the error signal for the carrier $\epsilon_0(\omega)$ and for the sidebands $\epsilon_s(\omega)$:

$$\begin{aligned} \epsilon_0(\omega) &\propto \sum_{i=1,4,7} 2A_i A_{i+1} \text{Im}[S_{i+1}] \\ \epsilon_s(\omega) &\propto \sum_{i=1,2,3,7,8,9} A_i^2 \left. \frac{d|R(\omega)|^2}{d\omega} \right|_{(\omega=\omega_i, \Delta=0)} \end{aligned}$$

where

$$S_i = R(\omega_{i-1})R^*(\omega_i) - R(\omega_i)R^*(\omega_{i+1}). \quad (4)$$

3. Experimental apparatus

The block scheme of the diagonal cavities test bench is shown in figure 3. In the following we describe the different parts of the apparatus: laser source, injection optics, test cavities and readout electronics.

3.1. Laser source

In order to achieve the desired long term stability of the geometrical parameters of the gyroscope, a stable laser source with good spectral purity is needed. The reference frequency is provided by an iodine stabilized He-Ne laser. Typical output power of this systems is of the order of 100 μW . Since for our application a higher power is required, following the approach of [19], we implemented a light amplifier based on injection-locking, so that the spectral purity of the reference laser is transferred to a 10 mW diode laser.

3.1.1. Reference laser The iodine-stabilized He-Ne laser is a master-slave system itself and it was provided by the Institute of Laser Physics of Novosibirsk, Russia. Here a He-Ne laser (slave) is phase-locked, with an offset frequency of 8 MHz, to another He-Ne laser (master). The frequency of the master laser is locked to one of 14 vibro-rotational optical transitions of molecular $^{127}\text{I}_2$ contained in a fused silica intracavity cell. The control system is based on the third derivative detection of the saturated absorption signal.

The principle of the stabilization is to keep the laser wavelength resonant to the chosen transition by adjusting the length of the cavity. To this purpose the master laser has two piezoelectric transducers acting on the position of the two cavity mirrors. A fast PZT is used to compensate the fast fluctuations and apply a 5 kHz modulation for the third derivative detection and a slow PZT compensates the thermal and mechanical frequency drifts. The relative Allan deviation of this laser reaches the level of 10^{-11} at 100 s.

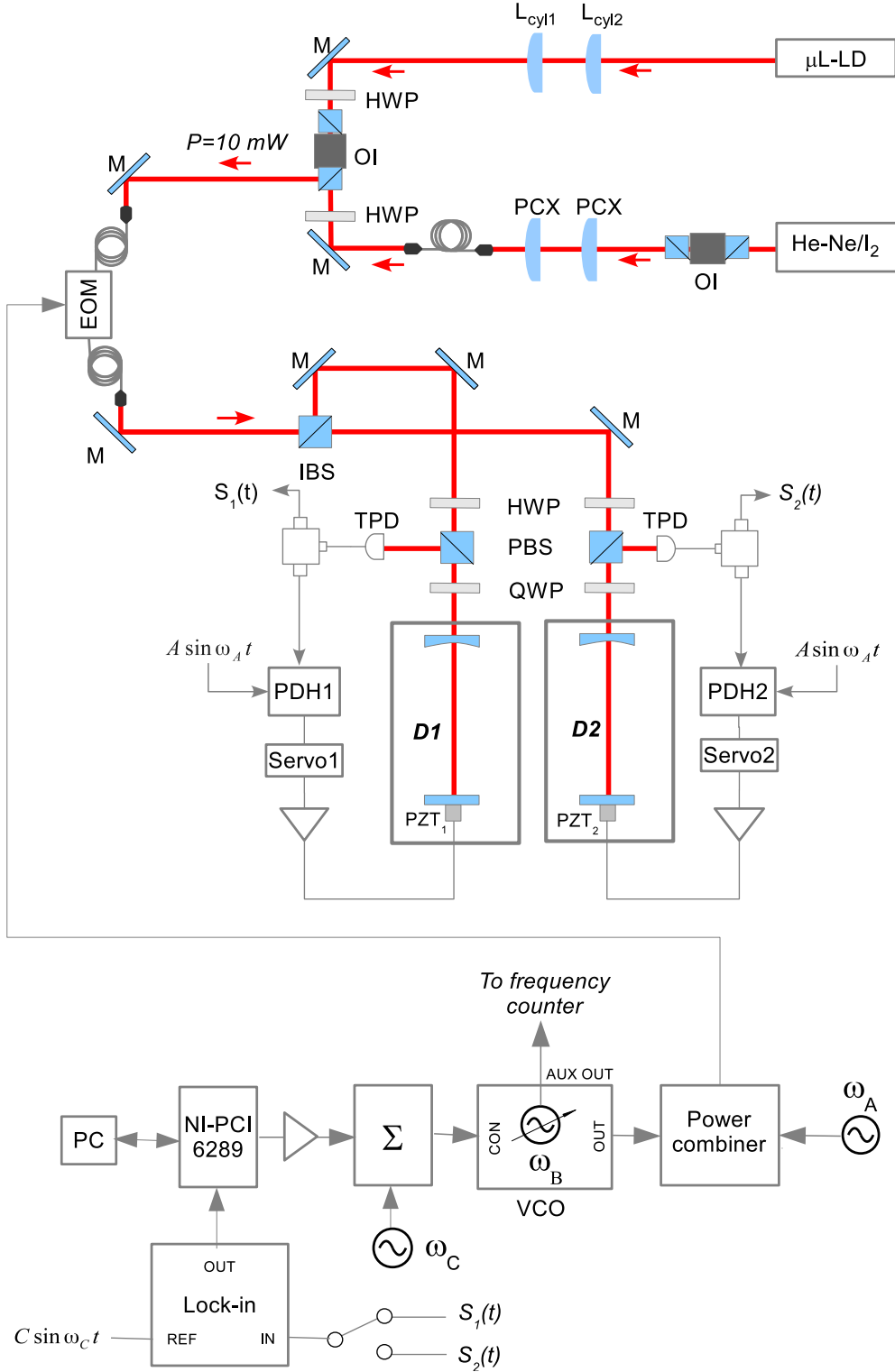


Figure 3. Block scheme of the experimental apparatus. The upper part represents the optical breadboard with the injection locking setup. $\mu\text{L-LD}$: micro-Lens Laser Diode, M: Mirror, HWP: Half Wave Plate, QWP: Quarter Wave Plate, PBS: Polarizing Beam Splitter, IBS: Intensity Beam Splitter, OI: Optical Isolator, VCO: Voltage Controlled Oscillator, TPD: Transimpedance Photodiode, PDH: Pound Drever Hall detection stage.

3.1.2. Probe laser The light source, acting as slave oscillator in the injection locking setup, is a diode laser emitting at 633 nm. Commercially available chips at this wavelength are quite unreliable since they are at the limit of the wavelength range of red diode lasers. As a consequence, in most of the cases their performances degrade very rapidly in time and typically require to be cooled down to several degrees below zero. Following [19] we selected the microlens-coupled diode laser chip (Blue Sky Research PS010). This system operating at 4.5°C provides a single-mode output power of about 10 mW resonant with the He-Ne transition. The chip is mounted inside a cylindrical aluminium assembly. A Peltier element and a thermistor are used to stabilize the temperature junction within 1 mK. The integrated microlens provides a nearly circular and Gaussian distribution of the intensity profile, making it possible to easily achieve a good mode matching with the He-Ne master laser.

3.2. Injection locking

The light of the reference laser is fiber coupled and injected into the slave laser through the output port of an optical isolator, as described in [19]. The typical master laser power entering the isolator is $P_{inj} \sim 100 \mu\text{W}$. The scheme of the injection locking setup is presented in the upper part of figure 3. To get an efficient coupling, the spatial mode of the injected beam must match the mode of the slave laser. We make use of a couple of cylindrical lenses in order to mode match the two lasers. The tuning of the free-running diode laser frequency is obtained by adjusting the junction temperature (coarse) and current (fine). At diode laser working point $(T, I_{diode}) = (4.5^\circ\text{C}, 65 \text{ mA})$ we obtain a typical locking-range [20] of $\sim 10 \text{ GHz}$.

3.3. Ring laser diagonal cavities test-bench

A drawing of the mechanical assembly of the Fabry-Pérot cavities used in our set-up is shown in figure 4. Two cavities are mounted onto an optical table equipped with a pneumatic isolation system. The two resonators consist of two spherical mirrors mounted on tip-tilt holders rigidly connected by Invar spacers. A piezoelectric translator is attached to the output mirror in order to scan the cavity and implement the feedback correction to the cavity length. The piezoelectric transducer is a hollow cylinder 20.3 mm in length and diameter, 1.5 mm in thickness. The measured mechanical response is 2.2 nm/V (full range voltage: 700 V). While the output mirrors have equal nominal specifications, the reflectivity of the input mirrors are different, leading to different cavity quality factors. In figure 5 is reported the transmission of a ring laser supermirror when operated at normal incidence. The parameters of the two Fabry-Pérot resonators are summarized in table 1.

3.4. Readout electronics

Before entering into the two cavities, the injection-locked diode laser beam is phase-modulated with a combination of 3 frequencies as reported in equation 4, with $\omega_A = 30 \text{ MHz}$, $\omega_B \simeq 680 \text{ MHz}$, $\omega_C = 33 \text{ kHz}$. This is done by means of a single fiber-coupled electro-optical phase modulator (EOM, Model PM635, Jenoptik). The reflected light from each cavity is detected with a transimpedance photodiode. The photodiode output is split in two lines using a power divider. One output of the power divider is demodulated at ω_A according to the standard Pound-Drever-Hall scheme in order to obtain an error signal $\epsilon_0(\omega)$ for the carrier lock. The second output of the divider is phase-detected by a digital lock-in amplifier (EG&G 7260)

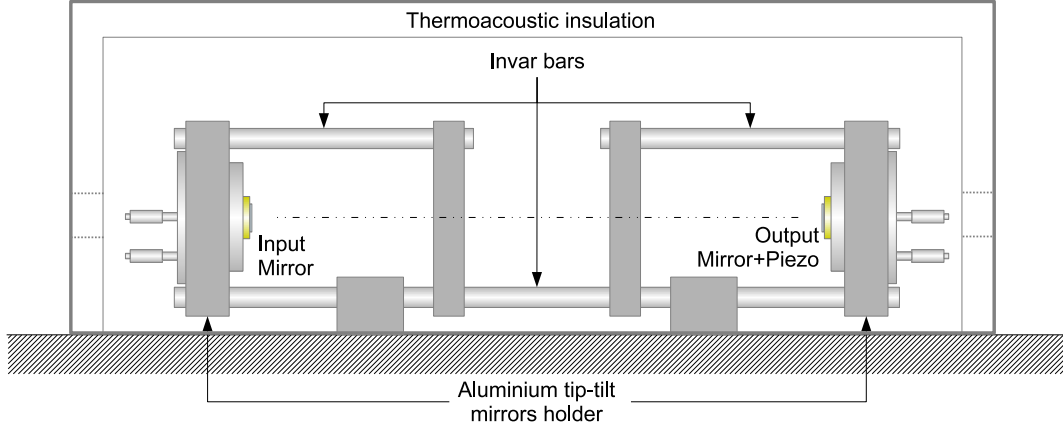


Figure 4. Mechanical assembly of the Fabry-Pérot cavities used in the test experiment.

Parameter	Cavity1	Cavity2
R	4 m	4 m
D	$\simeq 1.32$ m	$\simeq 1.32$ m
FSR	$\simeq 113$ MHz	$\simeq 113$ MHz
Ψ_R	$\simeq 0.27$	$\simeq 0.27$
r_{in}^2	0.988	0.997
r_{out}^2	0.999	0.999
\mathcal{F}	504	2094
$FWHM$	225 kHz	54 kHz

Table 1. Optical parameters for the two resonators emulating the diagonals of a ring laser of $1.32/\sqrt{2} \sim 0.93$ m in side length.

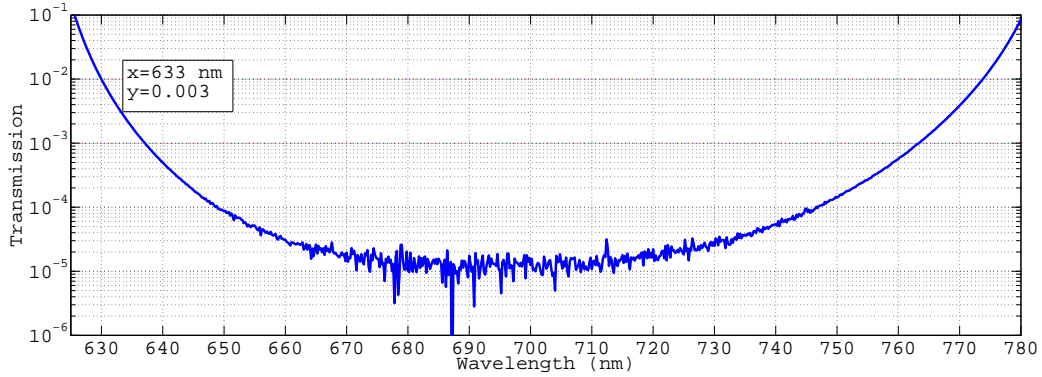


Figure 5. Measured transmission spectrum for the input mirror of cavity 1.

referred to the modulation frequency ω_C . The amplitude of the reflected signal in phase with the modulation is used as error signal $\epsilon_S(\omega)$ for the sideband lock. Simulated and measured error signals are shown in figure 6 and figure 7 respectively.

4. Results

The two error signals ϵ_0 and ϵ_S are used to apply corrections respectively to the cavity PZTs and to the VCO generating the sidebands at $\omega_B \sim 6 \cdot FSR$. The feedback loops controlling the cavity mirrors displacement are two analog PI circuits with a bandwidth of about 1 kHz.

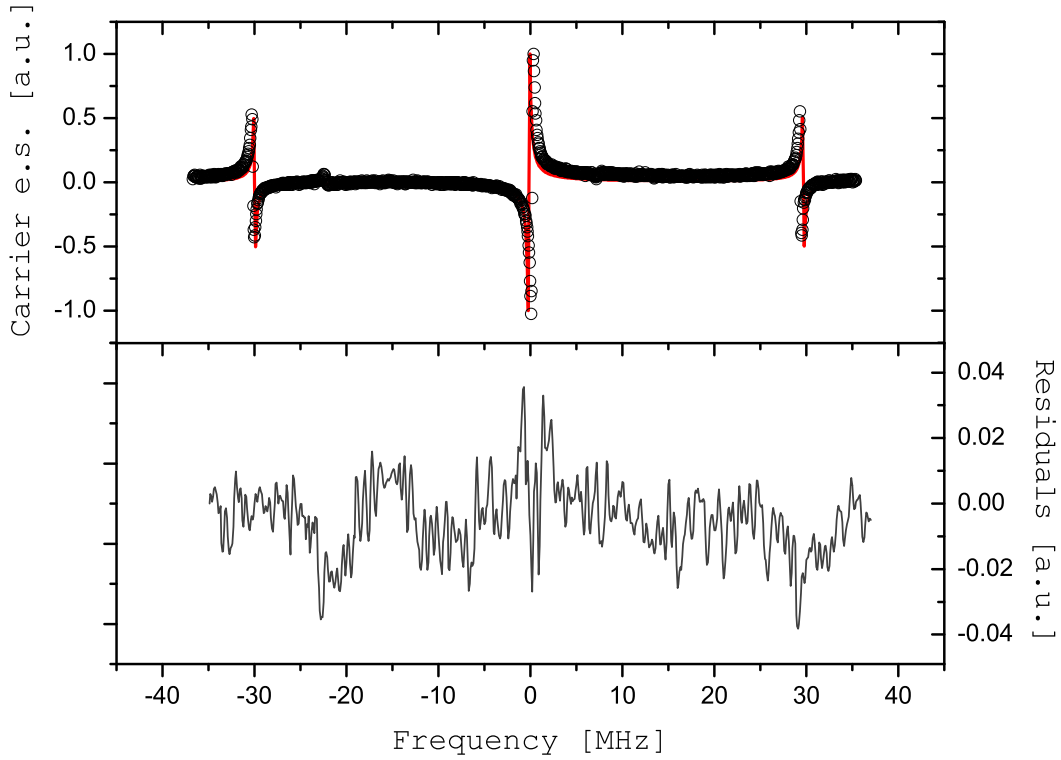


Figure 6. Above: simulated (continuous red line) and measured (black circles) carrier error signal versus ω_0 , when $\omega_B \sim 6 \cdot FSR$. Modulation parameters: $\alpha = 0.7$, $\omega_A = 30$ MHz, $\beta = 1$, $\omega_C = 33$ kHz, $\Delta \sim 200$ kHz. Sweep time: 100 ms. Below: residuals of the measured signal from the simulated one.

The residual noise in the error signal expressed as equivalent displacement noise for the two stabilized cavities is shown in figure 8.

In figure 9 we report the acquisition of the feedback signals controlling the two cavity lengths. In the same graph is also shown the room temperature variation. A drift of the cavity mechanics is clearly visible in addition to the effect of thermal expansion. Once the two cavity resonance frequencies are locked to the laser carrier frequency, the error signals $\epsilon_{S(1,2)}$ are processed by a LabView program running on a PC equipped with a DAQ board (NI-PCI 6289). The system locks the VCO frequency alternatively to the 6-th harmonic of FSR_1 or FSR_2 at about 680 MHz. As one FSR is determined with the required resolution, the VCO locks to the resonance of the other cavity. This is obtained simply by unlocking the loop, switching the error signal from $\epsilon_{S(1)}$ to $\epsilon_{S(2)}$ and re-lock. This operation is repeated periodically to verify the two numbers $n_{1,2} = \omega_0 / (2\pi \cdot FSR_{1,2})$. The auxiliary output of the VCO is connected to a microwave frequency counter (HP5343A) and the frequency counts are acquired by the PC via GPIB interface. A gaussian fit on the distribution of the frequency counts provides the frequency mean value of the sample f_c and its standard deviation σ . An example of dynamic resonance frequency estimation for the two cavities is presented in figure 10.

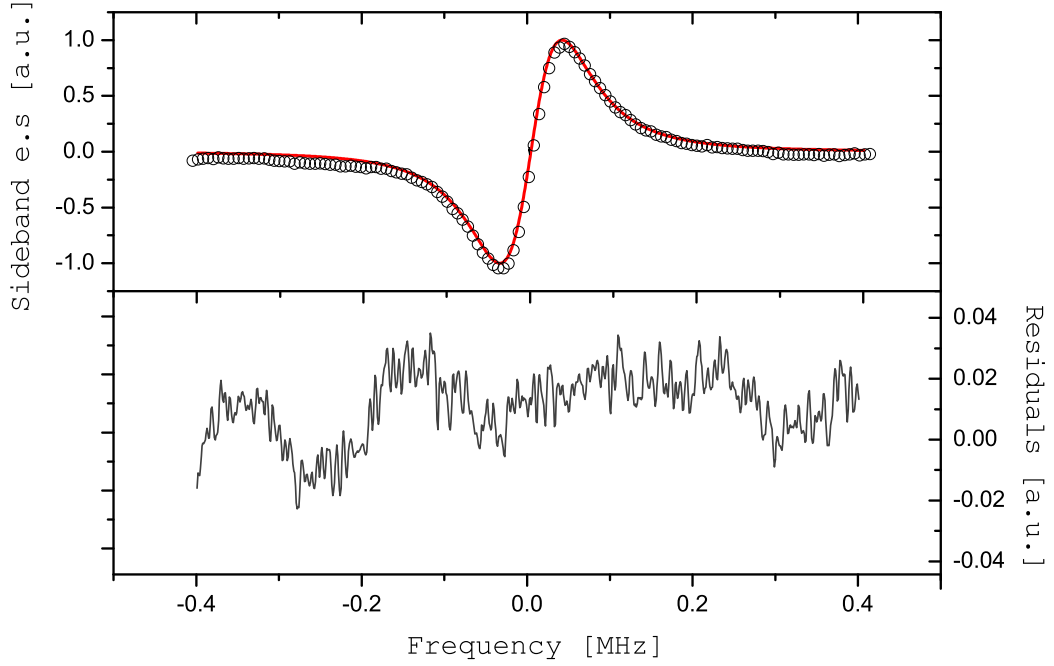


Figure 7. Above: simulated (continuous red line) and measured (black circles) sideband error signal versus ω_B . Modulation parameters: $\alpha = 0.7$, $\omega_A = 30$ MHz, $\beta = 1$, $\omega_B = 680$ MHz, $\omega_C = 33$ kHz, $\Delta \sim 200$ kHz. Lock-in sensitivity: 10 V/1 mV, lock-in time constant: 640 μ s, sweep time: 5 s. Below: residuals of the measured signal from the simulated one.

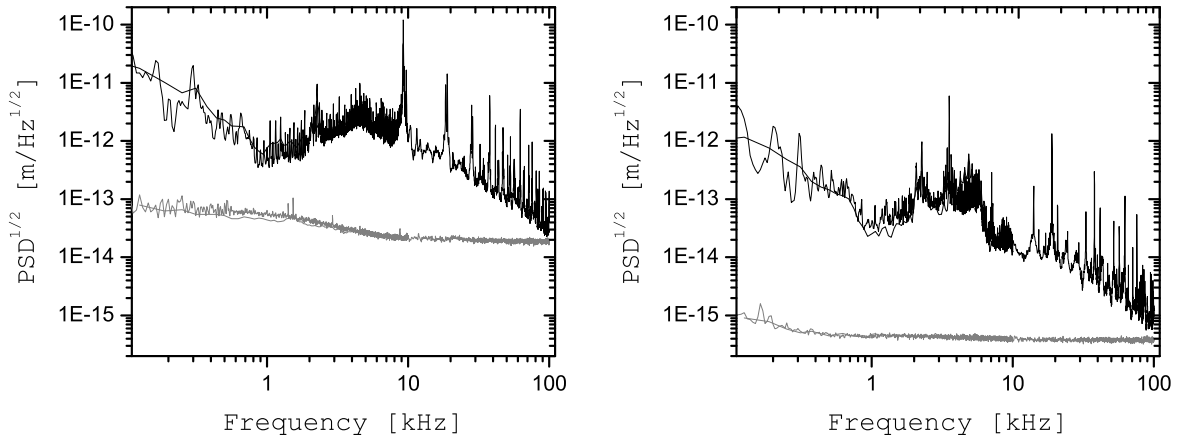


Figure 8. Power spectral density of the residual displacement noise for the cavity 1 (left) and cavity 2 (right). Gray lines are the electronic noise levels measured out of resonance.

4.1. Length-difference measurement

Let's consider now the accuracy on the estimate of the length difference between two cavities having the same kind of mirrors. In this case, the phase correction terms in equation 2 are equal and the mode number difference n_D :

$$n_D = \frac{\omega_0}{2\pi} \cdot \left(\frac{1}{FSR_2} - \frac{1}{FSR_1} \right)$$

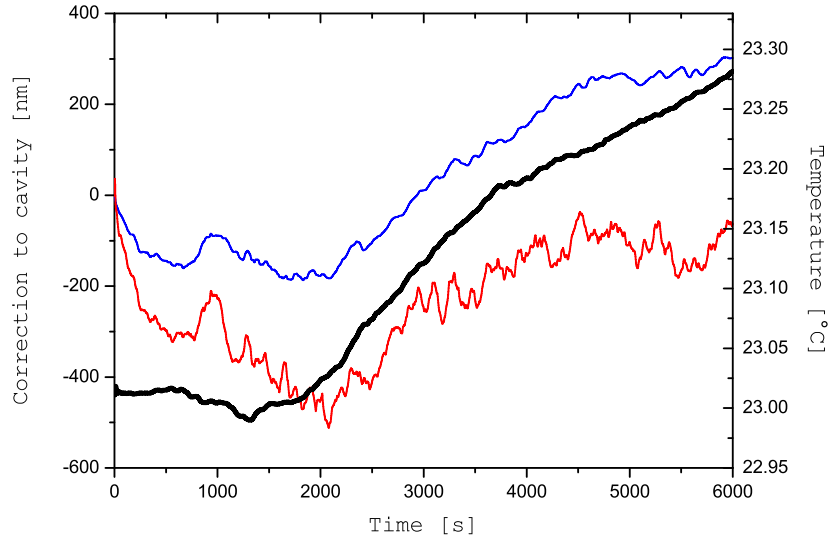


Figure 9. Thin traces: closed-loop corrections to the cavities output mirror positions (blue line: cavity 1; red line: cavity 2). Thick trace: room temperature.

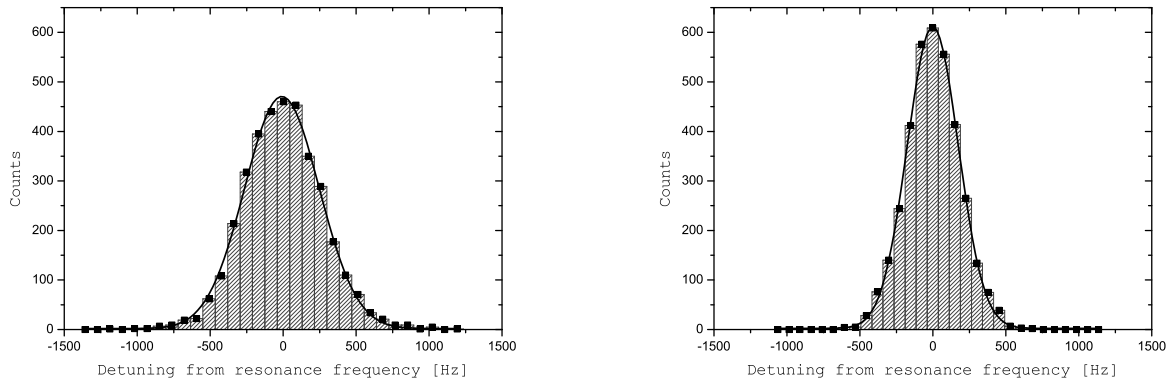


Figure 10. Frequency counting distribution for the measurements of $6 \cdot FSR$. Total measurement time: 140 min. Frequency counter gate-time: 1 s. From the gaussian fits on the two distributions we obtain an estimate for the central frequency f_c and the standard deviation σ . Cavity 1 (left): $f_{c1} = 681211560$ Hz, $\sigma_1 = 230$ Hz, $\chi_R^2 = 0.997$. Cavity 2 (right): $f_{c2} = 680000798$ Hz, $\sigma_2 = 140$ Hz, $\chi_R^2 = 0.998$

should be an integer number. Figure 11 shows the distribution of n_D obtained from the measurements of figure 10. The estimated mean value of the mode number difference is $n_D = 7427.4 \pm 1.6$. The obtained uncertainty on n_D sets the error on the length difference estimation to $\delta D = (\lambda/2) \cdot \delta n_D \sim 500$ nm.

5. Conclusions

We presented an experimental technique for the absolute length stabilization of the diagonal cavities formed by the opposite mirrors of large-frame He-Ne square ring lasers. As laser source for probing the cavities, a 10 mW diode laser injection locked to an Iodine stabilized He-Ne laser has been developed. In typical operational conditions the system has a locking range of

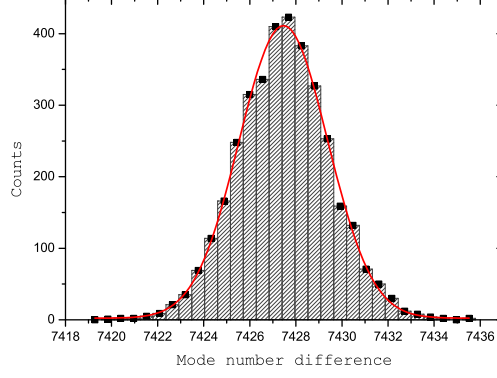


Figure 11. Distribution of the mode number difference n_D obtained from the distribution of figure 10. From the gaussian fit ($\chi_R^2 = 0.997$) we obtain $n_D = 7427$ and $\sigma_{n_D} = 1.6$

several GHz and a power amplification of the order of 100. The large value of the locking range makes the system operation very robust against diode laser frequency drifts. The detection system makes use of a single EOM driven with three different modulation frequencies, allowing to lock the two cavities to the laser carrier and also to perform a detection of an integer multiple of the two free spectral ranges. We demonstrated a stable locking of the resonance frequencies of the two cavities to the same reference laser, providing a length stabilization at the level of 1 part in 10^{11} at 100 s, and the determination of the two FSRs to few parts in 10^7 . The error in the measurement of the cavity length difference with our set-up is of 500 nm. We foresee the application of this technique to the GP2 prototype. For this ring laser the diagonal cavities will be contained inside a vacuum chamber and will have an expected finesse of 10^3 . In a temperature controlled laboratory, operating with a modulation frequency $\omega_B \sim 1$ GHz (EOM bandwidth cut-off) we expect to be able to determine the mode number difference with an uncertainty less than one. In this case the error on the length difference between the two diagonals will be ultimately limited by the uncertainty on the laser wavelength.

Appendix A.

Here we report some definitions recalled in the text. The reflection transfer function it is defined as:

$$R(\omega) = \frac{\sqrt{r_{in}^2 r_{out}^2} (e^{i\omega/FSR} - 1)}{1 - r_{in}^2 r_{out}^2 e^{i\omega/FSR}}, \quad (\text{A.1})$$

where r_{in} , r_{out} are the amplitude reflection coefficients for the input/output mirror, FSR is the cavity free spectral range and ω the laser frequency.

The amplitude and the frequencies of the nine components of the modulated laser field are listed in table A1.

The components of the reflected intensity oscillating at ω_A comes from the interference of the three groups of modes centered respectively at $\omega_0 (= \omega_5)$, $\omega_0 + \omega_B (= \omega_8)$, $\omega_0 - \omega_B (= \omega_2)$:

$$\begin{aligned} I_r^{\omega_A} \propto & 2E_0^2 \{ A_1 A_2 \{ \text{Re}[S_2] \cos \omega_A t - \text{Im}[S_2] \sin \omega_A t \} + \\ & + A_4 A_5 \{ \text{Re}[S_5] \cos \omega_A t - \text{Im}[S_5] \sin \omega_A t \} \\ & + A_7 A_8 \{ \text{Re}[S_8] \cos \omega_A t - \text{Im}[S_8] \sin \omega_A t \} \} \end{aligned} \quad (\text{A.2})$$

k	A_k	ω_k
1	$J_1(\alpha)J_1(\beta)$	$\omega_0 - (\omega_B + \Delta \sin \omega_C t) - \omega_A$
2	$-J_0(\alpha)J_1(\beta)$	$\omega_0 - (\omega_B + \Delta \sin \omega_C t)$
3	$-J_1(\alpha)J_1(\beta)$	$\omega_0 - (\omega_B + \Delta \sin \omega_C t) + \omega_A$
4	$-J_1(\alpha)J_0(\beta)$	$\omega_0 - \omega_A$
5	$J_0(\alpha)J_0(\beta)$	ω_0
6	$J_1(\alpha)J_0(\beta)$	$\omega_0 + \omega_A$
7	$-J_1(\alpha)J_1(\beta)$	$\omega_0 + (\omega_B + \Delta \sin \omega_C t) - \omega_A$
8	$J_0(\alpha)J_1(\beta)$	$\omega_0 + (\omega_B + \Delta \sin \omega_C t)$
9	$J_1(\alpha)J_1(\beta)$	$\omega_0 + (\omega_B + \Delta \sin \omega_C t) + \omega_A$

Table A1. Definition of the nine amplitudes and frequencies of the electric field spectral components.

where S_i is given by equation 4.

For small values of $\frac{\Delta}{\omega_B}$, the component of detected intensity oscillating at ω_C is written as:

$$\begin{aligned}
 I_r^{\omega_C} \sim E_0^2 & \left[A_1^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_1, \Delta=0)} + A_2^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_2, \Delta=0)} \right. \\
 & + A_3^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_3, \Delta=0)} + A_7^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_7, \Delta=0)} \\
 & \left. + A_8^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_8, \Delta=0)} + A_9^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_9, \Delta=0)} \right] \Delta \sin \omega_C t.
 \end{aligned} \tag{A.3}$$

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References

- [1] Schreiber K U and Wells J-P R 2013 Large ring lasers for rotation sensing *Rev. Sci. Instrum.* **84** (041101)1-26 doi:10.1063/1.4798216
- [2] Stedman G E 1997 Ring-laser tests of fundamental physics and geophysics *Rep. Prog. Phys.* **60** 615 doi:10.1088/0034-4885/60/6/001
- [3] Stedman G E *et al* 1995 T violation and microhertz resolution in a ring laser *Opt. Lett.* **20** (3) 324–326
- [4] Cooper L and Stedman G E 1995 Axion detection by ring lasers *Phys. Lett. B* **357** 3
- [5] Scully M O, Zubairy M S 1981 Proposed optical test of metric gravitation theories *Phys. Rev. A* **24** 4
- [6] Stedman G E, Schreiber K U, Bilger H R 2003 On the detectability of the Lense–Thirring field from rotating laboratory masses using ring laser gyroscope interferometers *Class. Quant. Grav.*, **20** 2527-2540
- [7] Chaboyer B and Henriksen R N 1988 Gravitational radiation observations with an orbital ring laser gyroscope *Phys. Lett. A*, **132** 391-398
- [8] Bosi F *et al* 2011 Measuring gravitomagnetic effects by a multi-ring-laser gyroscope *Phys. Rev. D* **84** (122002)1-23 doi:10.1103/PhysRevD.84.122002
- [9] Schreiber K U *et al* 2009 The large ring laser G for continuous Earth rotation monitoring *Pure Appl. Geophys.* **166** 1485-1498 Birkhäuser Verlag ISSN 0033-4553 doi:10.1007/s00024-004-0490-4
- [10] Schreiber K U, Gebauer, A, Wells J-P R 2013 Closed-loop locking of an optical frequency comb to a large ring laser *Opt. Lett.* **38** 3574 - 3577 doi:10.1364/OL.38.003574

- [11] Schreiber K U, Gebauer A, Wells J-P R 2012 Long-term frequency stabilization of a 16m2 ring laser gyroscope *Opt. Lett.* **37** 1925-1927 doi:10.1364/OL.37.001925
- [12] Belfi J *et al* 2012 A 1.82 m² ring laser gyroscope for nano-rotational motion sensing *Appl. Phys. B* **106** 271-281 doi:10.1007/s00340-011-4721-y
- [13] Beverini N *et al* 2014 Measuring general relativity effects in a terrestrial lab by means of laser gyroscopes *Las. Phys.* **24** 074005
- [14] Belfi J *et al* 2013 Absolute control of the scale factor in the GP2 laser gyroscope: toward a ground based detector of the Lense-Thirring effect *EFTF/IFC-21-25 July 2013, Prague* 795-798 doi:10.1109/EFTF-IFC.2013.6702268
- [15] Araya A *et al* 1999 Absolute-length determination of a long-baseline Fabry-Pérot cavity by means of resonating modulation sidebands *Appl. Opt.* **38** 2848-2856 doi:10.1364/AO.38.002848
- [16] Aketagawa M *et al* 2010 Free Spectral Range measurement of Fabry-Pérot Cavity using Frequency Modulation *IJPAP* **11** 851-856 doi:10.1007/s12541-010-0103-3
- [17] Aketagawa M *et al* 2011 Measurement of a Free Spectral Range of a Fabry-Pérot cavity using frequency modulation and null method under off-resonance conditions *Meas. Sci. Technol.* **22** 025302 doi:10.1088/0957-0233/22/2/025302
- [18] Hood C J, Kimble H J and Ye J 2001 Characterization of high-finesse mirrors: Loss, phase shifts, and mode structure in an optical cavity *Phys. Rev. A* **64** 033804, DOI: 10.1103/PhysRevA.64.033804
- [19] Vainio M, Merimaa M and Nyholm K 2005 Optical amplifier for femtosecond frequency comb measurements near 633 nm *Appl. Phys. B* **81** 1053-1057 doi:10.1007/s00340-005-1988-x
- [20] Siegman A E 1986 *Lasers University Science Book* cap.29 ISBN 0-935702-11-5